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LINEAR APERTURE CRITERIA FOR A STANDARD CELL LATTICE

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Among the parameters which need to be specified in the design of the SSC are the cell length (l_c) and the coil diameter (d_c). The quality of the magnetic field of the bending magnets, reflected in both small multipole coefficients and a small sigma for these coefficients, improves as d_c increases. Similarly the smaller β resulting from a decreased l_c results in an increased admittance (for a constant size beam pipe). Unfortunately increasing d_c and/or l_c would result in increasing the cost of the SSC. It is not only desirable but necessary to understand how the performance of an accelerator depends on d_c and l_c so that we can have confidence that the SSC when built will work. It should be straightforward to know whether or not an accelerator, once built, works or not. There are however no well accepted tools or criteria for deciding whether or not a proposed design of an accelerator will provide acceptable performance. This is particularly so in the case of accelerators built of superconducting magnets in which the magnetic field is determined by the current distribution in the coils.

To understand the basis of the criterion adopted for this analysis let us begin with the motion of a particle in a linear lattice, i.e. a lattice composed only of linear elements, viz quads, bends and drift regions. Further we will assume no coupling between the x and y planes of the particle motion, i.e. we have only normal quadrupoles and no skew quadrupole components. The solution to this problem is well known viz

$$u(s) = d\sqrt{\frac{\beta}{\beta_{max}}}\cos(\phi+\delta) \qquad (u \equiv x \text{ or } y)$$

where $\phi = \int_0^s \frac{ds}{\beta(s)}$

$$u' = \frac{-d}{\sqrt{\beta\beta_{max}}}\left[\alpha\cos(\phi+\delta) + \sin(\phi+\delta)\right]$$

$$\alpha = \frac{-1}{2}\beta'$$

The constant d can be expressed in terms of the initial condition:

$$d = \sqrt{\frac{\beta_{max}}{\beta_0}}\left[u_0^2 + (\alpha_0 u_0 + \beta_0 u_0')^2\right]^{1/2}$$

We can also, for the linear machine, express the invariant as

$$\frac{\epsilon}{\pi} = \gamma u^2 + 2\alpha u u' + \beta u'^2 \qquad \gamma \equiv \frac{1+\alpha^2}{\beta}$$

$$= \frac{d^2}{\beta_{max}}$$

Let us assume that a particle with initial condition u_o, u_o is tracked around a lattice and the quality ε is calculated after each complete turn. If the lattice is linear we will calculate the same value of ε after each turn. If there are non-linear elements in the lattice, e.g. chromaticity sextupoles or high order multipoles in the bending magnets, ε in general will vary from turn to turn. The spread in ε after a number of turns is a measure of how closely the lattice under study approximates a linear machine. In the following discussion the spread in ε will be defined as $\delta\varepsilon = \sqrt{\langle\varepsilon^2\rangle - \langle\varepsilon\rangle^2}$ where the averages are computed for 100 turns. If $\delta\varepsilon/\varepsilon$ is small then we may feel confident that the lattice will behave in a manner which approximates a linear machine. The magnet multipoles are of course not the only source of problems in an accelerator once it is built. Other factors, such as alignment errors, will reduce the actual performance.

Since the object of this study was to provide information on the behavior of a lattice as a function of d_c and l_c a very simple model of an SSC was constructed with 481 standard cells. The circumference of the model SSC was allowed to vary as l_c changed.

In every case the phase advance per cell was 60°. In addition to the principal focusing quads two families of trim quads and of chromaticity sextupoles were incorporated into the lattice to allow one to vary the tune and chromaticity of the lattice in both planes. Between the focusing quads 1,2 or 4 non-linear kicks are placed in the bending dipoles, one kick per dipole. High order multipoles are assigned to each dipole. The moments are gaussian distributed with a sigma for each moment assigned either from the values in the report of the Reference Design Study Group or from values supplied by the Central Design Group for magnet design "D" and for a superferric magnet. When varying the diameter of a magnet (d_c) the n^{th} moment ($n = 1$ for the quadrupole moment) is assumed to scale as $(d_o/d_c)^{n+1/2}$ with $d_o = 4$ cm. In the following results all systematic errors and also all random quadrupole (both normal and skew) moments are set to zero.

Tracking of the particles was done with TEVLAT. A particle was started at x_o, y_o, $x_o' = y_o' = 0.0$ and the initial value of ε_x/π and ε_y/π were calculated. It was then tracked for 100 turns and at the end of each turn ε_x/π and ε_y/π were again calculated. After the tracking was completed the quantities

$$R = \sqrt{(\epsilon_x/\pi)^2 + (\epsilon_y/\pi)^2}$$

$$\delta R = \sqrt{\sigma^2(\epsilon_x/\pi) + \sigma^2(\epsilon_y/\pi)}$$

were computed. The parameters x_o, y_o, d_c and l_c were varied and the process was repeated again. The set of values for $\delta R/R(C, d_c, l_c) = Q$

$$\left(C = \left[\frac{\beta_{max}}{\beta_{ox}} x_o^2 + \frac{\beta_{max}}{\beta_{oy}} y_o^2 \right]^{1/2} \right)$$

is then fit to the form

$$Q = k \left(\frac{l_c}{l_o}\right)^{\alpha} \left(\frac{d_o}{d_c}\right)^{\beta} C^{\gamma}$$

where k, α, β, γ are determined by least squares fitting.

The coefficients k and γ are sensitive to the ordering of the magnets in the lattice. It is not expected that α or β should be sensitive to the ordering. It is necessary in comparing the results from different cases and for interpreting the final results to know the range of k and γ resulting from different arrangements of the magnets. To study this effect 30 different permutations of the magnets in the lattice were used and Q was calculated for each permutation for several values of r. The results for the various cases studied can be found in Table I. The value of k apparently scales as $1\sqrt{N}$ where N is the number of kicks per cell. For the different permutations we find $\sigma(k)/k \simeq$ 0.3 where $\sigma(k)$ is the variance of k about the mean. γ is apparently independent of the number of kicks.

The variation of Q for the different permutations is very large. The ratio of $\sigma(k)/k$ ranges from 0.27 to 0.38 for the cases listed in Table I. A plot of k vs γ for the configuration with 4 kicks per cell (Figure 1) shows that the ratio of the maximum value of k to the minimum value is >3. A possible explanation for this could be the variation of the smear with the azimuthal position as we go around the ring. A preliminary calculation shows that, for a particular ordering of the magnets, the smear (Figure 2) varies by only ~20%. This result suggests that the spread in k observed by permuting the magnets reflects the importance of the magnet ordering and not the point where we start the tracking.

The value of β is independent of the number of kicks and is found to be β = 2.56 with an uncertainty of ~0.05. This value is the same when calculated using the BRD and the CDG moments. It is also the value one should expect if $Q = \delta R/R$ were due to sextupole moments in the magnets.

The interim report of aperture Group B (April 22, 1985) of the SSC Aperture Task Force has proposed that the smear be defined as

$$S \equiv \Delta C/C$$

$$C = \sqrt{a^2+b^2}$$

$$\frac{a^2}{\beta_{max}} = \epsilon_x/\pi \qquad \frac{b^2}{\beta_{max}} = \epsilon_y/\pi$$

and ΔC is the radius of the circle about C including all the values for C_i calculated in the tracking. Their criteria is that the smear be less than 10% at C = 7 mm. This definition of ΔC does not correspond exactly to our definition of $\delta c/c$ (= 1/2 Q). If we ignore the difference between $\Delta C/c$ and $\delta c/c$ then using our fit to Q we can calculate for different values of C, d_c and l_c corresponding to a smear of ±10%. The curves in Figure 3 are plotted for k = 0.038 corresponding to the 5 kicks/half cell and k = $\langle k \rangle + 2\langle \sigma_k \rangle$. These conditions correspond to one of the criteria in the report of the Aperture Task Force.

The results up to now have been obtained with zero chromaticity in the lattice. The chromaticity sextupoles, being non-linear elements, contribute to the smear. As the length of the cell l_c decreases the effect of the chromaticity sextupoles on the smear increases because the strength of the sextupoles, and hence their contribution to the smear, increases while the smear due to the random multipoles is decreasing. The contribution of the sextupoles is to increase the smear at small l_c and to decrease α when the results of the calculation at small l_c (= 100 m) are included in the fitting.

We conclude the criteria for smear in the report of April 22, 1985 can be satisfied with a cell length of 200 m and a coil radius somewhat greater than 5 cm. Work is underway to further refine these results.

Table I

Ensemble Averages

d_c = 4 cm $\quad l_c$ = 213.36 m

No random quadrupole moments

Conditions		$k \pm \sigma(k)$	$\gamma \pm \sigma(\gamma)$
BRD moments	1 kick/half cell	$(4.76 \pm 1.79)\times 10^{-2}$	1.29±0.12
	2 kicks/half cell	$(3.63 \pm 0.98)\times 10^{-2}$	1.35±0.14
	4 kicks/half cell	$(2.59 \pm 0.83)\times 10^{-2}$	1.32±0.12
CDG moments	"D" 1 kick/half cell	$(4.45 \pm 1.71)\times 10^{-2}$	1.30±0.13
CDG moments, superferric magnet	1 kick/half cell	$(2.90 \pm 0.92)\times 10^{-2}$	1.39±0.11

Acknowledgement

A number of the calculations whose results are summarized in this report were performed by A. Russell. His contribution to this study is gratefully acknowledged.

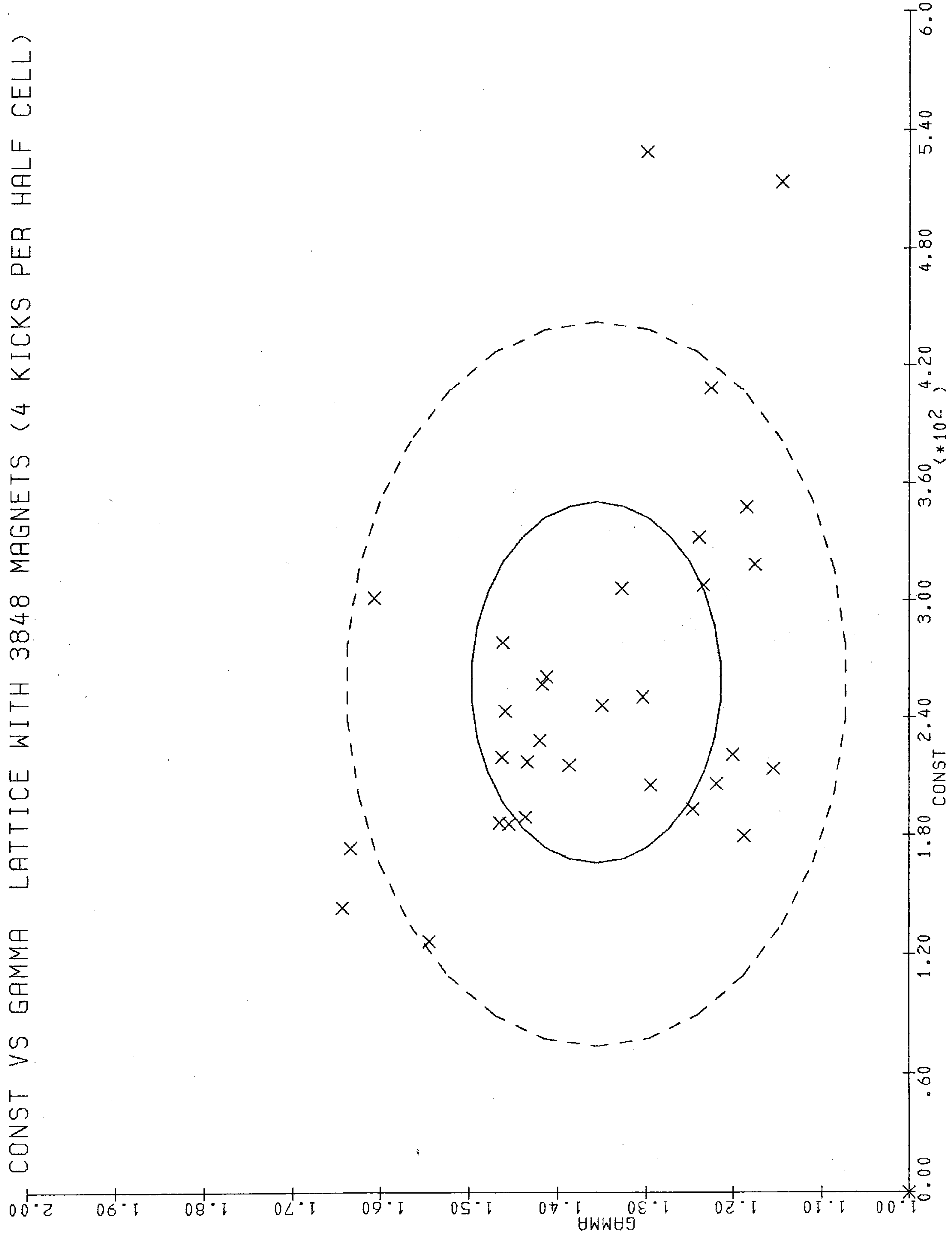
It is a pleasure to thank Don Edwards and acknowledge his insight and direction which contributed greatly to this work.

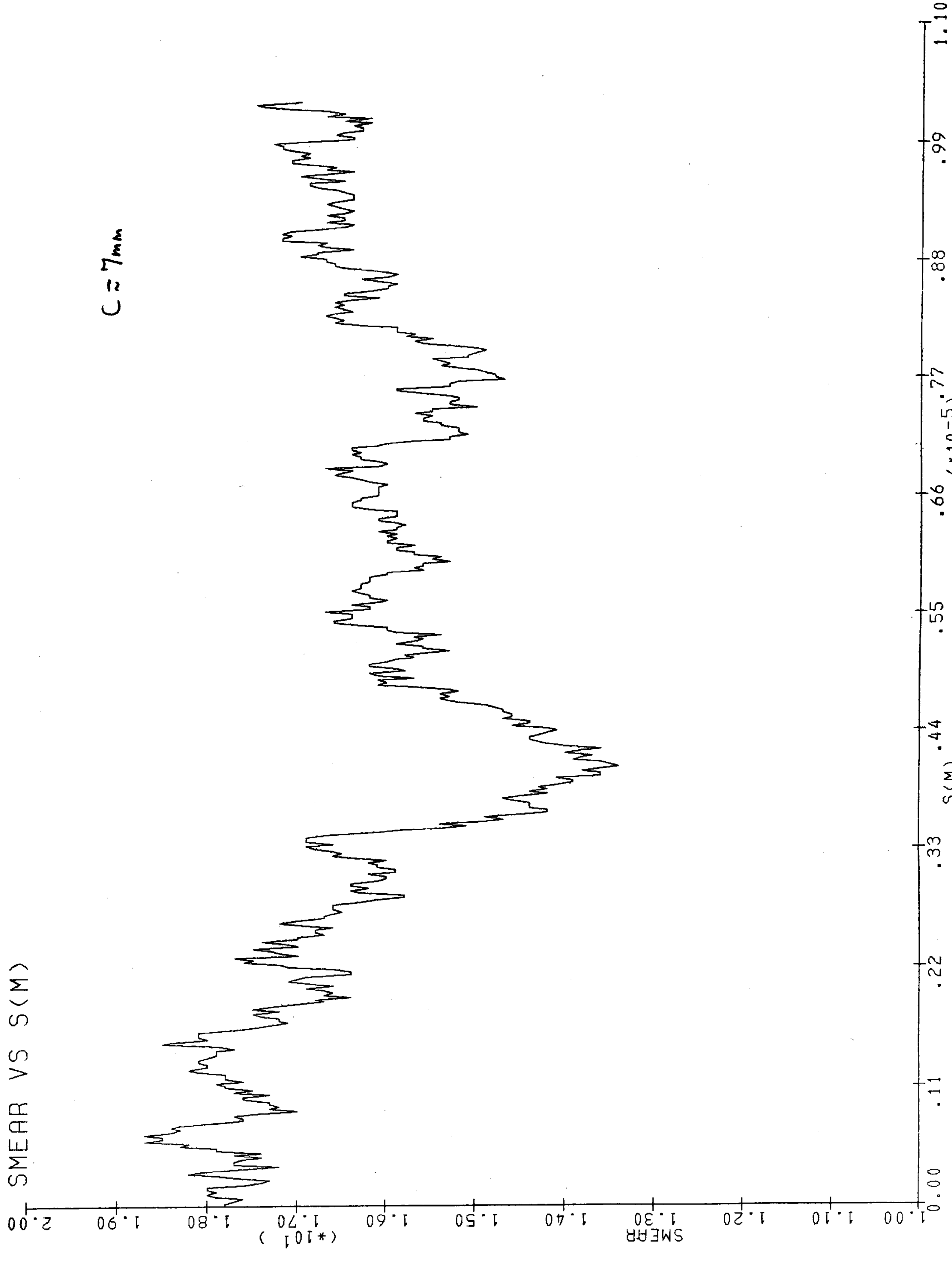
Figure Captions

Figure 1 Plot of k vs γ for 31 permutations of the magnets, 4 kicks per half cell

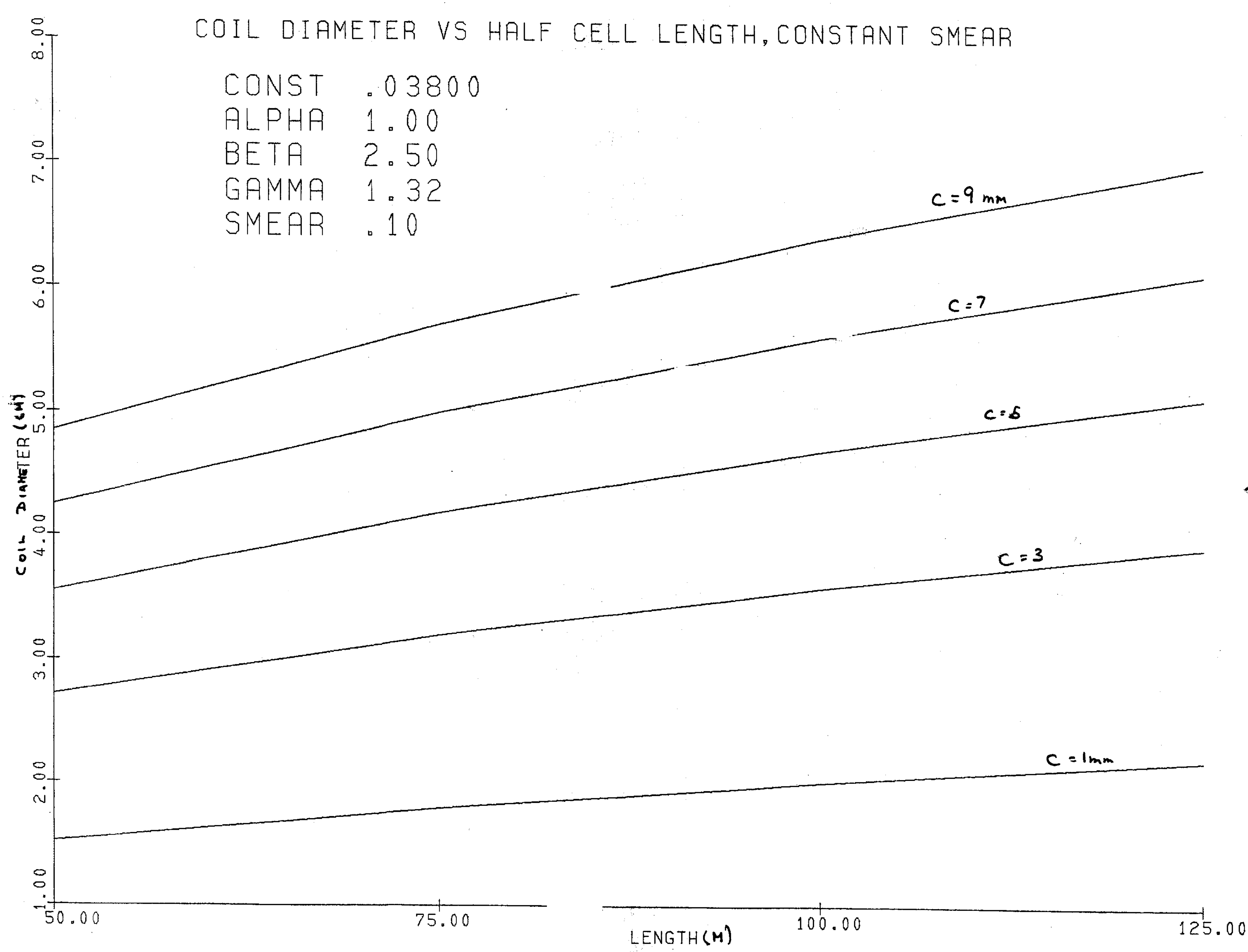
Figure 2 Plot of $\delta c/c$, vs. s, c = 7 mm, 4 kicks per half cell

Figure 3 Coil diameter vs half cell length, smear = 10%


CONST VS GAMMA LATTICE WITH 3848 MAGNETS (4 KICKS PER HALF CELL)
CONST
(*10^2)
0.00
.60
1.20
1.80
2.40
3.00
3.60
4.20
4.80
5.40
6.00
GAMMA
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00



SMEAR VS S(M)
C ≈ 7mm
SMEAR
(*10¹)
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.00
S(M)
(*10-5)
0.00
.11
.22
.33
.44
.55
.66
.77
.88
.99
1.10



COIL DIAMETER VS HALF CELL LENGTH, CONSTANT SMEAR
CONST .03800
ALPHA 1.00
BETA 2.50
GAMMA 1.32
SMEAR .10
C = 9 mm
C = 7
C = 5
C = 3
C = 1mm
COIL DIAMETER (cM)
LENGTH (M)
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
50.00
75.00
100.00
125.00

12

570